The Development of New Analytical Techniques

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The analysis and design of the structure is only one of the activities needed for the final construction of a building; the titles of the other papers in this discussion meeting help to place structural analysis in its proper context. Relative importance may be judged by cost; the cost of the structure of a high-rise office block, of a concert hall, or of a block of flats might be only one quarter of the total cost of the building. It is well to remember that some extraordinary new development in structural analysis, leading to a saving of 20% of the cost of the structure, might save only 5% overall. Five per cent is worth saving, of course, and the smallness of the figure is in any case no excuse for the structural engineer to rest content with existing methods.

However, there are also other savings to be made by the structural engineer which have nothing to do with structural analysis as such. The nature of these savings can be seen clearly with a construction which is almost entirely 'structure' rather than building; for example, a bridge. A change in the suggested site of a bridge may lead to savings which far outweigh any possible economies which might arise from a refined structural analysis. Once the best siting has been determined, then further fundamental economies may be made by the correct choice of form and of material; again, these are decisions which have nothing to do with the analytical techniques finally used for design.

Some of these same considerations apply to the design of an office block. The main material, for example, could be steel, perhaps with a reinforced-concrete core for a high-rise building; or it could be all reinforced concrete; or it could be steel used with reinforced concrete to form a composite construction. Again, the architect's basic floor plan can be modified by the engineer, perhaps to increase some spans and to decrease others; these modifications may be considered together with the choice of material. Once again, such overall planning of the building is likely to lead to a particular economic solution before any detailed thought is given to the actual structural design. These considerations may well lead to the conclusion that time and effort spent on refining the design calculations should be limited; this may be true, but it must not be concluded that a rough and ready design technique is all that is necessary. On the contrary, the designer should be aware at each stage of the accuracy of his calculations, so that he can put a more or less precise figure to the margin of safety of his final design.

Above all, he cannot be allowed to shelter behind the paradox that a really refined technique of structural analysis might be thought to be completely unnecessary. This paradox arises from the observation that if a structure is to be safe in practice, it cannot be too sharply designed; it must have a sufficient margin of safety against many extraneous factors, and this margin will in fact allow for some bluntness in design method. (To digress for a moment, the correct implication here is that an engineering disaster is generally not due to some small failure in design or construction technique; if a catastrophe occurs, then *prima facie* it will be due to some major error in conception or execution.)

Now the kind of decision which is concerned with the overall concept of the building is made by a senior engineer on the basis of his experience. The decisions are not arbitrary, but neither do they seem explicable in analytical terms. The engineer will make more or less elaborate design studies to help him reach his decision, but the final step seems to be into the dark. Engineering may be thought of as the art of making decisions on the basis of insufficient information, in the sense that, using a mathematical analogy, there are too few equations to determine uniquely the values of the unknowns. But this analogy, while helpful as far as it goes, can be misleading. After all, since some decision *is* reached, there must be some train of thought which can be discovered or deduced; any other conclusion merely insults the senior designer, by suggesting that his decisions rest on the toss of a coin. In a sense it might be that the most important advance to be made in the art of structural design lies in the determination of the thought processes of our senior designers. Can these processes be understood? If they can, then can students be trained so that they may eventually exercise their minds in the same way?

These are questions which can perhaps be answered obliquely, by considering in detail some aspects of structural design. First, the kind of traditional training we provide in our universities is no complete preparation for the kind of creative activity we have in mind. The classical equations of structural mechanics are taught, or, more strictly, perhaps, the applications of those equations are taught; in any case, it is the applications that loom large, and the basic equations remain unquestioned. This type of training is assumed to provide a firm foundation for the design of any type of superstructure but it is, in fact, a dead process, which decomposes quickly into rigid sets of design rules, of the sort which used to be guarded with ritual in the lodges of the medieval masons, and which are now accorded the biblical status of Codes of Practice.

An extreme example of this kind of *rigor mortis* is the belief that most problems in structural engineering were solved in the nineteenth century. The argument goes that because the methods developed required excessively long calculations, they did not provide a solution in the engineering sense; but they do now, when the calculations can be made on a high speed computer. What is meant here is that, by the end of the nineteenth century, it was possible to set up complex equations for structural analysis; now, as then, there is a tendency for the engineer to believe that once he has formulated his problem in mathematical terms, then that problem is effectively solved. The results coming off a computer are accepted without hesitation, but they are really only as good as the original assumptions; the assumptions necessarily made by the engineer in getting his problem into a manageable state are very rarely questioned.

Now there is evidence on this point available for at least one type of structure, the steel frame. Whether the structural equations are solved roughly or exactly, by hand or by a modern computer, the work that John Baker did, forty years ago, while working for the Steel Structures Research Committee, showed that the results bear virtually no relation to the actual behaviour of a steel-framed structure. The reasons are not far to seek, although this is not the present point; they lie in the assumptions made about the way the members of the structure are connected to each other and to the foundations. The fact is that, far from having been solved in the last century, most problems in structural engineering are still being tackled. It is only very recently that the steel frame, which surely is the simplest of all structures, is at last yielding to a proper analysis by the use of plastic theory. And even so the solutions are limited; the problem of the continuous column, for example, and others which appear in the design of high-rise buildings, require much further work.

It is very dangerous to think that, because a nineteenth-century set of equations can at last be solved by a twentieth-century machine, a real design problem has been mastered. This kind of danger attaches to the use of a really powerful new analytical technique, that of finite elements. The method opens up a whole new range of problems for which the classical equations can be solved, at least approximately; but once again, the solution of the equations does not necessarily mean that any real advance has been made in structural design. All too often the designer will be presented with a set of numbers, say the values of elastic stresses at various points in the structure, which may or may not have any real bearing on the assessment of the strength and safety of the structure. Further, it may be that the idealization of the structure into a patchwork of finite elements will itself suppress the most critical types of behaviour; local instability leading to overall catastrophic collapse may not only be entirely ignored, it may be actually inhibited.

All this is really a plea for the intelligent rather than the automatic use of computers in structural design. No one would expect a designer who keeps his nose firmly in a Code of Practice, applying routine rules without understanding, to have any insight into actual structural behaviour. There is a danger that this will be exactly paralleled by the use of computers; the computer itself may be credited with a store of rule-of-thumb experience, but its operators will be increasingly ignorant.

A second example of non-advance in the field of structural design is afforded by a study of the sort of pressures placed on the designer by the necessity to compete in the market place. This *is* a necessity; an academic structural engineer may be able to produce a masterpiece of analytical technique, but this will be of no use at all if the resulting structures prove to be more costly than those produced by conventional theory. The designer must be willing, in fact, to have his ideas tested by practice; indeed, he should have a broad enough education (and this must be discussed more fully in a moment) to do some of the testing himself.

However, there is a possibility that the whole design process becomes inverted, with the actual design made in the market place, and subsequently checked by the structural engineer. In practical terms, the situation can be reached in which the problem of structural design is handed over to production engineers, in the hope of achieving maximum economies; the resulting product, which may bear little resemblance to a conventional structure, is then checked for safety by a Code of Practice which is really not relevant. Put in the most simplistic way, the Code may assume that a twenty-storey building has a steel or concrete frame, whereas the building being checked may be assembled out of prefabricated units and be unframed.

The universities are well aware that, in this sort of context, they must give a somewhat broader training to engineering students; they must not be allowed to specialize in the single field of structural design. Since it is no good designing something if it cannot be built, then some knowledge is needed of material properties, of metal-forming processes, and of general fabrication techniques; sophisticated instrumentation demands some knowledge of electronics; if the engineer is to use the computer to the best advantage, then he must know something of numerical techniques; thermodynamics may well be useful, and so on. There will not be much room left in such a syllabus for any structural engineering, so that the next logical step is almost inevitable, and the universities can make a final simplifying move; all students can be taught the same basic course, irrespective of the final branch of engineering in which they will practice.

The next stage in such an educational programme is to widen the syllabus still further, to institute graduate courses for those subjects for which there is obviously no further room at undergraduate level. Courses can be added on topics such as operational research, business management, economics, sociology, and psychology; courses, in short, in total technology. The fact that there is still no room for any deeper study of actual topics in structural engineering need cause no dismay; designers of the future may not be able to advance technical reasons for the collapse of their structures, but they will at least be able to place individual engineering failures in a precise sociological context.

Engineering education is so important in any discussion of technical developments that more must be said later. In the meantime, after these negative and perhaps sour comments, some more positive speculation may be made about possible advances in structural analysis.

One way of getting to grips with a structural problem is to make a model and, perhaps surprisingly, there seems to be room for advance in this field. Some of the advance is in fact in concept, in determining more precisely what it is the designer is about when he makes and tests a model. For example, it is clear that a proposal for an indirect model test, which relies on the validity of the elastic reciprocal theorem, has already assumed a certain design philosophy: in this case, that the elastic behaviour is the prime criterion for design. It is not so clear that a direct model test may also suffer from the making of prior assumptions. As an example, a model of a concrete shell roof may be made in order to determine bending effects near the edges, and strain gauges may be grouped in critical regions so that these effects can be measured. It may be, however, that the value of a local bending moment in a shell roof is of no real significance in its overall design, and the resulting structure may then be designed upon the wrong sort of information. In a sense, of course, this difficulty is inherent in all practical scientific work; it is impossible merely to *observe* a test without any preconceptions, or indeed to design a test without having some hypothesis that one wishes to prove or disprove. The sticking of a strain gauge is not done at random; in fact, the very appearance of a strain gauge shows at once that this type of test is really an analogue solution of certain equations that have already been formulated. Once again, the results of the test, that is, the solution of the equations, will be relevant if the basic equations are a fair reflexion of reality, and not otherwise.

Certain types of model test, however, seem to be fairly free of vice. Direct observations of collapse loads of ductile steel frames are measurements of a quantity which is of obvious relevance to design (which is not to say that a designer can blindly assume for any particular building that ultimate strength will be the overriding design criterion; he may well have to consider also the elastic behaviour under working load). Similarly, the results of direct wind-tunnel tests on models of bridge decks to confirm their aerodynamic suitability are of great and immediate use to the designer.

Laboratory testing, whether on models or full-scale, is one of the fundamental activities in a programme of applied research. To illustrate by an example – the growing use of composite structures, perhaps associated with a particular type of fabrication technique, leads to a standard pattern of investigation: basic tests, followed by a hypothesis to be tested more critically in the laboratory, leading to a more or less empirical theory which can finally be reduced to sets of design rules. This is a pattern being followed at the moment for the use, in the inelastic range, of composite steel and concrete for building frames; a less clear-cut project, in which considerable advances are possible, concerns the interaction of cladding, partitions and floors with the frame of a tall building.

These topics, however, are examples of possible advances that may be made by using existing techniques, whether of design or of construction. For more fundamental progress, the basic equations of structural mechanics must be examined, to see where possible advances might arise. The equations are only three in number; they make certain statements first of all about the equilibrium of the structure and of its parts; secondly, about the properties of the material being used, such as measures of its elasticity and its ultimate strength; and finally, about the deformations of the structure. Each of these three classes of statement can be examined to try and predict possibilities for new techniques of analysis.

At first sight there seems little to be said about equilibrium, which appears to say no more than that a structure must stand up. But the question becomes more complex if the formulation of the equilibrium equations is looked at more closely. The engineer is used, for example, to drawing a centre-line diagram for a building frame, and to satisfying the equilibrium equations for this centre line and not for the real building. This is an example of a way in which a practical problem can be simplified for solution, and the simplification in this case works very well, in the sense that good estimates are obtained of the primary structural forces. (It must not be forgotten that secondary forces may govern the design locally; real connexions must be provided between beams and columns, and the forces in connexions may be very different from those given by the centre-line model.)

It is natural to make this same sort of simplification when dealing with shells, rather than frames, and to imagine that the material of a shell roof, for example, which may well be very thin, is concentrated in a central surface. This leads to simple equations for membrane stress resultants, but it turns out that the idealization is not nearly so good as it is for the two-dimensional frame. In the first place it is often found that the simple solution does not apply near the boundaries of the shell. Secondly, and perhaps more importantly, the tyranny exercised by a given central surface forces solutions which are at best limited, and which may in fact be completely irrelevant to the problem being studied; this is the same sort of phenomenon that was observed by the Steel Structures Research Committee when it attempted to correlate conventional elastic theory for a bare steel frame with actual observed behaviour.

As an example, the earthquake of 986 led to the collapse of one quarter of the main dome of St Sophia; the three-quarter dome that was left, spanning about 30 m, remained standing. Conventional simple shell theory is completely at a loss when dealing with a three-quarter masonry dome; such an irregular structure does not fit the pattern of the basic equations. However, once the straight jacket of the concept of the central surface is loosened, it is in fact easy enough to demonstrate that a three-quarter dome of a small but finite thickness is indeed stable.

The way this is done is to use a cutting technique on the structure, and to consider the basic equilibrium of separate portions. Such a cutting technique is quite old, but has been reintroduced recently as a development of plastic theory, and it looks very promising as a new key to the understanding of structural behaviour. Plastic theory itself is the most important modern innovation in structural design, and has come about as a result of a critical examination of the second of the three master statements, that of material properties. The Steel Structures Research Committee went as far as it could with a description of structural behaviour in purely elastic terms. It was the abandonment of these ideas, and the investigation of behaviour when the limiting yield stress of a ductile material is reached, that led to the development of the tools of plastic design. These tools have been sharpened to the point where they can be used with confidence in the design of steel frames where strength is the important design criterion.

Plastic methods have now become absorbed by the present generation of designers, and the next move is just starting, based on the realization that it is the proper method of design for a whole range of structures, not only those made from mild steel. Indeed, structures made from any material which is ductile enough to be used safely in practice may be analysed by plastic theory. With the use of plastic theory will come the development of cutting techniques, and of a whole expertise of structural design based on equilibrium and material properties with little or no attention paid to deformations in appropriate cases.

Consideration of material properties is almost certain to lead in the near future to the abandonment of design based on guaranteed values of those properties. Instead of a guaranteed yield stress a characteristic strength may be quoted, together with a maximum standard deviation, and a design can be made on a probabilistic basis, using the relevant portions of established probability theory. The loading on the structure can also be regarded stochastically, and finally the whole design may be made to give a small, but finite, probability of failure. Once again, the mathematical model which leads to such a design must be examined carefully by the engineer; since the required probability of failure is presumably very small, calculations will be made far along the tail of any distribution of the relevant quantities, whether that distribution is normal or skew. It may be that the quantities cannot in fact be extrapolated to such a tail; as a crude example, the distribution of weights of vehicles is cut off by legislation prohibiting very large trucks.

Advances will be made in the use of anisotropic materials, which are reinforced to give desired properties, whether of strength or stiffness, in certain directions. For building frames, the problems are likely to centre round the connexions. At sections remote from discontinuities, that is, at sections where St Venant's principle may be assumed to apply, design will be straightforward, but it is not so easy to dispose anisotropic material in regions close to boundaries. A bolt hole may be drilled with relative impunity in mild steel, but may require considerable reinforcement if the material is anisotropic. Much experience has been gained, of course, with a very widely used anisotropic material, reinforced concrete. Other types of reinforced material will require the development of corresponding technologies, involving both theory and laboratory experiment.

Examination of the third equation of structural mechanics, that of deformation, at once indicates a large field in which much work is needed. Problems of deflexions of tall buildings, for example, still require solution. For this type of structure the deformation and equilibrium equations may interact; the particularly simple situation in which equilibrium equations are satisfied for the undeformed state of the structure may no longer lead to a valid solution. Column loads can become largely eccentric, and lead to premature instability; the control of overall deflexions is again a field that will repay study, and there are several obvious paths open for investigation.

The general problem of instability is perhaps the most rewarding and it is here that the most significant advances in structural analysis are likely to be made. At one end of the range, the growth of large deflexions due to the development of plasticity is now reasonably well understood. Similarly, the ideas of simple elastic buckling are part of the vocabulary of the designer, although in fact the design of a continuous column in a building frame, or the thin web of a plate girder, or the stiffened diaphragm in a box girder, still require further work. Indeed, practical design problems involving instability are likely to be so complex that the importance of direct model and full-scale tests cannot be overemphasized.

Perhaps the most interesting problems lie in a middle range where instability is neither a purely elastic nor a purely plastic phenomenon, and where the behaviour is much affected by precise conditions of loading, by erection procedures, by residual stresses due to welding, and so on. A pin-ended column, for example, could be designed to have a slenderness such that its critical stress for buckling was equal to the yield stress of the material, so that the plastic squash load and the elastic instability loads were reached simultaneously. It is in precisely this sort of range that ignorance of the behaviour of more complex structures is largest, both theoretically and experimentally. In fact there are good theoretical reasons for expecting a large experimental scatter if two modes of failure are likely to occur simultaneously. There are also good reasons for perhaps avoiding such an apparently optimum situation; failure, when it does occur, is likely to be not a gentle matter, but catastrophic.

This leads on to the real problem which faces the structural engineer, which is that of design, not of analysis. The examples discussed so far have presupposed a structure whose behaviour must be determined; what is needed is a method which will, ideally, generate an efficient structure to fulfill certain functions, such as carrying specified loads with certain limiting deflexions. Constraints will be imposed on the form of the structure; for example, a building frame will usually have rectangular bays. Such constraints may be introduced as part of the design brief; others will be introduced by the designer immediately he starts to use a given design technique. It is usual, for example, to view a building frame as a two-dimensional structure, and to design it that way; perhaps a more efficient structure would result if the building were viewed three dimensionally. However, techniques for analysis and design in three dimensions are not well developed.

Not only is there room for advance here in an attempt to produce a more efficient structure; there is also room for the development of better optimization processes. The simple minimizaton of total material consumption is obviously a fairly crude way to design, although this is the field that has so far been explored most thoroughly, mainly due to the fact that plastic theory can be used to give direct answers. Definite theoretical advances have been made here, and there is now some measure of what one type at least of the ideal structure should look like. The introduction of more complex cost functions, involving perhaps considerations of transport, plant hire, alternative fabrication techniques, and so on, will almost certainly lead to design based upon the results of a search, inevitably undertaken by computer. Here the advances will be not in structural theory, but in the actual search techniques; a promising development for structural analysis seems to lie in the application of dynamic programming.

These are some of the ways in which structural analysis may advance, but who will make the advances? Are they to be made by fully qualified engineers already working in industry, or by young graduates in the universities? If they are to be made at all, then it is important to ensure that the training programme for structual engineers is of the right kind. The existing programme may be exemplified by a study of a hypothetical chief designer of today, aged say 55 or 60. He will have graduated in the mid-thirties, and then started on his practical training in the drawing office, later moving on to the site. In the drawing office he may well have spent three months drawing rivet holes, and today he could tell, merely by glancing at the board of a junior draughtsman in his office, whether a riveted connexion was well or badly designed. Unfortunately he is unable to exercise this hard-won experience; no riveted connexions are being designed in his office. Instead, his draughtsman may be detailing a welded joint; in assessing the fatigue strength of this joint, the chief designer may be in no better position than his newest graduate trainee, and he may be in a worse position, if the young graduate has been properly taught, and if the senior designer has not kept up with current practice.

Developments have been sufficiently fast in recent years for the traditional experience of senior men to count for very little. Unfortunately, junior men are by definition inexperienced, and there is a very real danger that, in design offices staffed under the present educational and training system, there will in fact be no one capable of making an advanced design. In this context it becomes essential to review the traditional British system, in which half the training is in the university and half in industry.

First of all, the undergraduate must be taught nothing which approximates to detailed design; otherwise he will learn the modern equivalent of riveted connexions, which may be of immediate use but will be quickly out of date. Instead, the *theory* of structures must be taught as broadly and as deeply as possible. At the same time, despite the earlier sour comments in this paper, it is in fact more and more necessary that the engineer should have as wide a training as possible. With only a limited time, there is an inevitable conflict of aims, and, inevitably, engineers will obtain a first degree which reflects only an imperfect education. It is true that patterns of instruction at the university can be less rigid; seminars, programmed learning techniques, reading courses, project work, and so on, can be used to much greater effect. (New ways of teaching must be carefully directed, and care must be taken not to abdicate responsibility for what is taught; an open-ended project must not be too open ended. It would be clear to surgeons that they could not present a patient to a group of medical students as an open-ended project.)

The young graduate will have, however, only a sketchy knowledge of the techniques of structural analysis when he goes to be trained in industry; moreover, the days in which he might hope to learn how to design say a skyscraper, merely by working the office and on site, now seem to be past. It seems inevitable that the engineer must return to a university for postgraduate training, and a possible educational pattern for the future begins to emerge, in three parts rather than the traditional two which can develop fairly naturally, from the many postgraduate courses now existing. In order to become fully qualified and chartered, the engineer must have first a broad basic first degree; secondly, a period of professional training in industry; and thirdly, the equivalent of a Master of Engineering degree. This final degree would be awarded after really deep theoretical instruction in a specialized field, and would be the final step necessary for a professional qualification. Once it has been accepted that four years at the university are necessary, then there is room for some rearrangement of the timetable. The whole four years could, for example, be done before the period in industry. Alternatively, the basic degree could be awarded after two years at the university; the period in industry could then be followed by a one- or two-year Master's course.

However it is arranged, such an educational programme will demand that university staff maintain close contact with industry, preferably by working from time to time in the design office. Perhaps the single greatest contribution to the development of new analytical techniques of structural design will be made in this way; not by a significant theoretical advance, but by the exposure of practising engineers to postgraduate courses, and of professors to the rigours of practical design.